Sensitivity of spin-glass order to temperature changes

1993 J. Phys. A: Math. Gen. 26 L641

(http://iopscience.iop.org/0305-4470/26/15/005)

J. Phys. A: Math. Gen. 26 (1993) L641–L646. Printed in the UK

LETTER TO THE EDITOR

Sensitivity of spin-glass order to temperature changes

I Kondor†‡ and A Végső‡

† Bolyai College, Eötvös University, Budapest, Hungary
‡ Institute for Theoretical Physics, Eötvös University, Budapest, Hungary

Received 11 March 1993

Abstract. By a loop-expansion around Parisi's mean-field theory for an Ising spin-glass it is shown that the overlap of the magnetization patterns belonging to two different temperatures, T and T', vanishes to any order, $\overline{\langle s_i\rangle_T\langle s_i\rangle_{T'}}=0$, while the correlation overlap $\overline{\langle s_is_j\rangle_T\langle s_is_j\rangle_{T'}}$ calculated to first loop order (and, for technical reasons, for dimensions $d>8$ only) is found to decay exponentially, with a characteristic length $\sim(T-T')^{-1}$.

Although the two main theoretical approaches to the spin-glass (SG) problem, the replica field theory (Mézard *et al* 1987) and the phenomenological droplet theory (Bray and Moore 1987a, Fisher and Huse 1986, 1988), rest on substantially different physical pictures and lead in many respects to sharply conflicting predictions, in some important problems they do show a surprising degree of agreement. One of these is the sensitivity of the spin-glass order to changes in the external parameters. This sensitivity, sometimes referred to as the chaotic nature of the SG phase, consists, among other things, in a complete reorganization of the equilibrium magnetization patterns $\langle s_i\rangle$ by the slightest change in temperature or by the addition of an arbitrarily small external field. As a consequence, the overlap between two patterns belonging to two different temperatures T, T' vanishes for any $T\neq T'$:

$$\overline{\langle s_i\rangle_T\langle s_i\rangle_{T'}}\equiv\hat{q}_{T,T'}=0 \tag{1}$$

and that between patterns belonging to a finite field h and to zero field, respectively, vanishes for any h:

$$\overline{\langle s_i\rangle_h\langle s_i\rangle_0}\equiv\hat{q}_{h,0}=0. \tag{2}$$

Here $\langle\ldots\rangle$ means thermal averaging, and the overbar average over the (symmetrically distributed) random couplings. Equations (1) and (2) are to be contrasted with the fact that for strictly coinciding external parameters $\langle s_i\rangle^2$ is positive in the SG phase, by definition. Statements (1) and (2) have been known in replica mean field theory (MFT) for a long time ((2) is implied by a result in Parisi 1983, while (1) is quoted by Binder and Young 1986 from an unpublished work by Sompolinsky), but they also naturally follow from the underlying assumptions of droplet theory (Bray and Moore 1987b, Fisher and Huse 1986, 1988). Within this latter framework another aspect of SG sensitivity has also been pointed out: infinitesimal changes in the external parameters upset not only the patterns but also the correlations. While at a given T and $h=0$, the correlation overlaps like, for example, $\overline{\langle s_is_j\rangle\langle s_is_j\rangle}$ are generally believed to decay as a power of the distance between the sites i and j (another point on which the many valley and droplet theories agree qualitatively), the overlaps between correlation functions belonging to two different temperatures

$$C_{T,T'}=\overline{\langle s_is_j\rangle_T\langle s_is_j\rangle_{T'}} \tag{3}$$

or to a finite resp. zero field

$$C_{h,0}=\overline{\langle s_i s_j\rangle_h \langle s_i s_j\rangle_0} \tag{4}$$

are predicted to fall off exponentially, with coherence lengths $\xi_{TT'}$ and ξ_{h0} that diverge in the limits $T\to T'$ and $h\to 0$, respectively. In order to check whether these predictions are also borne out by the replica theory, one of the authors (Kondor 1989) calculated the correlation overlaps (3), (4) in a Gaussian approximation, i.e. neglecting interactions between fluctuations, and found that $C_{h,0}$ did indeed show the expected behaviour with a coherence length $\xi_{h,0}\sim h^{-2/3}$, but $C_{T,T'}$ remained power-like for any $T, T'<T_c(\xi_{T,T'}=\infty)$. While this sort of behaviour is not inconceivable, given (1), it is rather counterintuitive. In addition, it was proposed by Fisher and Huse (1988), and by Koper and Hilhorst (1988) that the sensitivity to control parameters may account for some of the effects seen in ageing experiments (Lundgren *et al* 1983, Alba *et al* 1987), and this raised the hope that measurements may perhaps discriminate between the two rival theories.

The purpose of the present letter is to calculate $C_{T,T'}$ beyond the Gaussian approximation and decide whether (i) the long range nature of $C_{T,T'}$ is a genuine feature (as it is for $T=T'$, Kondor *et al* (1992)), or (ii) $C_{T,T'}$ becomes short-ranged once interactions are taken into account, which would eliminate the conflict between the two theories on this particular point, or (iii) a negative gap, i.e. an instability, is generated which destroys the whole many valley theory. Our main result is that (ii) is the case and that in high enough dimensions the characteristic length associated with $C_{T,T'}$ goes as $\xi_{T,T'}\sim|T-T'|^{-1}$. The technique we use to establish this result is perturbative, and as such it is confined to the dimensionality range above the upper critical dimension, $d>6$. For technical reasons the first-order (one loop) result we derive is, in fact, valid only in the even more restricted range $d>8$. As will be explained below, in order to cover the range $6<d<8$ we should go to second order which, though feasible, would take about an order of magnitude more algebra. In order to go below $d=6$ we should abandon perturbation theory and resort to the renormalization group whose structure is, at the moment, poorly understood in the SG phase. All these limitations notwithstanding, there is very little doubt in our minds that the perturbative conclusion about the generation of a gap in the spectrum of $C_{T,T'}$ remains valid down to the (unknown) lower critical dimension.

Let us start by recapitulating some of the relevant mean-field results. We consider two independent copies of the same Sherrington–Kirkpatrick (SK) (1975) system, one at temperature T, the other at T', with the same set of Gaussian random couplings J_{ij}. The mean value $\overline{J_{ij}}$ will be taken zero, the variance $\overline{J_{ij}^2}=1/N$, so the critical temperature $T_c=1$. For T and T' both in the vicinity of T_c the application of the replica trick (Edwards and Anderson 1975) leads one to consider the following 'free energy' functional:

$$\mathscr{F}=-\lim_{n\to\infty}\frac{1}{n}\left\{\frac{1}{2}\sum_{\alpha,\beta}\tau_{\alpha\beta}Q_{\alpha\beta}^2+\frac{w}{6}\operatorname{Tr}Q^3+\frac{u}{12}\sum_{\alpha\beta}Q_{\alpha\beta}^4-\frac{y}{4}\sum_{\alpha\beta\gamma}Q_{\alpha\gamma}^2Q_{\beta\gamma}^2+\frac{v}{8}\operatorname{Tr}Q^4+\ldots\right\} \tag{5}$$

where

$$\tau_{\alpha\beta}=\begin{cases}\tau=(1-T^2)/2 & \alpha,\beta\leqslant n/2\\ \hat\tau=(1-TT')/2 & \alpha\leqslant n/2<\beta \text{ or } \beta\leqslant n/2<\alpha\\ \tau'=(1-T'^2)/2 & \alpha,\beta>n/2.\end{cases} \tag{6}$$

The coupling constants w, u, y, v all work out to be 1 for Ising-like spins ($s=\pm 1$). Yet it is expedient to keep them as essentially free parameters, for two reasons. (i) On the MF level the free energy $\mathscr{F}$ with a generic set of coupling constants corresponds to an SK-like system for soft spins, with the critical temperature and the couplings being determined by the various cumulants of the local spin distribution. (ii) Above the upper critical dimension the finite range model we wish to study can be mapped back onto MFT, with a new set of coupling constants.

The order parameters $Q_{\alpha\beta}$ are determined by the extremum condition

$$-\frac{\partial\mathscr{F}}{\partial Q_{\alpha\beta}} \equiv h_{\alpha\beta} = 2\tau_{\alpha\beta}Q_{\alpha\beta} + w(Q^2)_{\alpha\beta} + \tfrac{2}{3}uQ^3_{\alpha\beta} - yQ_{\alpha\beta}[(Q^2)_{\alpha\alpha} + (Q^2)_{\beta\beta}] + v(Q^3)_{\alpha\beta} = 0. \tag{7}$$

The solution is easily found by the following considerations: the upper left $n/2\times n/2$ block of $Q_{\alpha\beta}$ describes an ordinary SK spin-glass at temperature T, hence this block must be of the Parisi (1979) form. The same holds for the lower right $n/2\times n/2$ block with T replaced by T'. The off-diagonal blocks describe the overlaps between these two systems; at the extremum these overlaps must have a unique value: $Q_{\alpha\beta}\equiv\hat{Q}$, $\forall\alpha\leqslant n/2<\beta$. Indeed, if this is so, the off-diagonal blocks do not influence the solutions in the diagonal ones, because, according to (7), $\hat{Q}$ enters the equations for $Q_{\alpha\beta}$, $\alpha,\beta\leqslant n/2$ or $\alpha,\beta>n/2$ always in combinations carrying a factor n, hence they drop out in the replica limit. If, on the contrary, we built any structure into the off-diagonal blocks they would necessarily modify the solutions in the diagonal ones, which is clearly absurd: the act of comparing two systems at different temperatures cannot influence them. To find $\hat{Q}$ we work out (7) for $\alpha\leqslant n/2<\beta$:

$$\hat{Q}\{2\hat{\tau} + w(S+S') + \tfrac{2}{3}u\hat{Q}^2 + v(S^2+SS'+S'^2) - y(R+R')\} = 0 \tag{8}$$

where

$$S = -\int_0^1 Q(x)\,\mathrm{d}x = -\frac{\tau}{w} - \frac{2y+3v}{2w^3}\tau^2 + \ldots$$

$$R = -\int_0^1 Q^2(x)\,\mathrm{d}x = -\frac{\tau^2}{w^2} + \ldots$$

and similarly for S', R', with τ replaced by τ'. Obviously $\hat{Q}=0$ is a solution of (8) which immediately yields, in MF, the result (1). However, $\hat{Q}=0$ is not unique, additional roots come from the curly bracket in (8). To check the stability of the solutions is therefore vital. This is what we turn to now.

The spectrum of fluctuations is given by the eigenvalues of the Hessian

$$M_{\alpha\beta,\gamma\delta} = \frac{\partial^2\mathscr{F}}{\partial Q_{\alpha\beta}\partial Q_{\gamma\delta}}.$$

For $\hat{Q}=0$ the Hessian $M_{\alpha\beta,\gamma\delta}$ has a block-diagonal structure. The upper left ($\alpha<\beta\leqslant n/2$, $\gamma<\delta\leqslant n/2$) and lower right ($n/2<\alpha<\beta\leqslant n$, $n/2<\gamma<\delta\leqslant n$) blocks describe the fluctuations of the Parisi order parameters at temperature T and T', respectively which are known to be marginally stable (De Dominicis and Kondor 1983). The middle block ($\alpha\leqslant n/2<\beta$, $\gamma\leqslant n/2<\delta$) describes the fluctuations of the overlap $\hat{Q}$ about its zero average value. This block has the same structure as the matrix diagonalized by Kondor (1989), whence we know that its smallest eigenvalue is

$$\lambda_{\min}=\frac{1}{2}\left(\frac{v}{w^2}-1\right)(T-T')^2. \tag{9}$$

The sign of $\lambda_{\min}$ depends on v/w^2. For Ising-like spins $w=v=1$, so $\lambda_{\min}=0$, in agreement with Kondor (1989). The question arises, however, whether there are soft-spin distributions for which the ratio v/w^2 deviates from unity. A little reflection shows that by an appropriate rescaling of $Q_{\alpha\beta}$ one can always keep $w=v=1$, therefore the $\hat{Q}=0$ solution is always marginally stable in MFT.

A side remark is in order here: following Parisi (1979) it has become customary to keep only the u term of the quartic couplings which is the one responsible for replica symmetry breaking. This truncated model works perfectly in the standard, $T=T'$, situation, in that it reproduces all quantities correctly to leading order in τ (Kondor *et al* 1992). In the present case, with $T\neq T'$, we see however, that putting $v=0$ would produce a spurious instability.

The eigenvalues of the middle block of the Hessian are the poles of the Fourier transform of the correlation function (3) in Gaussian approximation. The zero eigenvalue just found means that for large distances $C_{TT'}$ falls off like a power. As for the other solutions of (8), coming from the curly bracket, we find that for $v/w^2=1$ (i.e. in MFT) they are the same as $\hat{Q}=0$ above, and when short-range corrections make $v/w^2>1$ they become unstable. We do not need to consider them any more.

Now we turn to the short-range corrections. The field-theoretic loop expansion around the Parisi solution has been applied to the standard $T=T'$ problem by De Dominicis *et al* (1991) and Kondor *et al* (1992). The technique we use is a straightforward extension of that explained in these papers. In particular, the effective Lagrangian governing fluctuations is formally the same as in equation (1) in Kondor *et al* (1992), with τ replaced by $\tau_{\alpha\beta}$ (and with the completely irrelevant s-coupling dropped), so we do not need to display it here.

The equation of state derived from that Lagrangian reads to first-loop order:

$$\begin{aligned} h_{\alpha\beta}+\frac{1}{z}\int\frac{\mathrm{d}^d p}{(2\pi)^d}\Big\{ & w\sum_{\gamma} G_{\alpha\gamma,\beta\gamma}(p)+2uQ_{\alpha\beta}G_{\alpha\beta,\alpha\beta}(p)\\ & +v\sum_{\gamma\delta}\left(Q_{\gamma\beta}G_{\alpha\delta,\delta\gamma}(p)+Q_{\gamma\delta}G_{\alpha\delta,\gamma\beta}(p)+Q_{\alpha\delta}G_{\delta\gamma,\gamma\beta}(p)\right)\\ & -y\sum_{\gamma}\left(Q_{\alpha\beta}G_{\alpha\gamma,\alpha\gamma}(p)+2Q_{\alpha\gamma}G_{\alpha\gamma,\alpha\beta}(p)+Q_{\alpha\beta}G_{\beta\gamma,\beta\gamma}(p)\right.\\ & \left.+2Q_{\beta\gamma}G_{\beta\gamma,\alpha\beta}(p)\right)\Big\}=0 \end{aligned} \tag{10}$$

where the propagators are given by

$$(G^{-1})_{\alpha\beta,\gamma\delta}=p^2\delta_{\alpha\beta,\gamma\delta}+M_{\alpha\beta,\gamma\delta}. \tag{11}$$

We have already noted that if the $\alpha\leqslant n/2<\beta$ components of $Q_{\alpha\beta}$ vanish then M becomes block diagonal. By (11), so does G. But then from the $\alpha\leqslant n/2<\beta$ components of (10) we immediately deduce that the $1/z$ terms give no contribution to the equation of state which, therefore, preserves its MF form $h_{\alpha\beta}=0$ ($\alpha\leqslant n/2<\beta$), from which $\hat{Q}$ can be factorized out again. Moreover, it is clear that the same argument applies to any order: from the assumption $\hat{Q}=0$ it follows that the fluctuations in the T and T'

systems decouple and this guarantees that $\hat{Q}=0$ is a solution indeed. Provided this solution is stable, $\hat{Q}\sim\overline{\langle s_i\rangle_T\langle s_i\rangle_{T'}}=0$ is, therefore, an exact statement.

Turning to the investigation of the fluctuation spectrum now we apply the trick of the large-p expansion explained in De Dominicis *et al* (1991) and Kondor *et al* (1992). The essence of the method is the trivial observation that in high enough dimensions the loop integrals are dominated by wavevectors which are large compared with any characteristic mass in the problem. The resulting loop-corrections will be free of any IR divergences and therefore can be absorbed into the coupling constants, whereby the loop-corrected theory is mapped back onto MFT. To get this mapping, it is sufficient to substitute the large-p expansion of the propagators (obtained by iteration directly from (11)) into (10) and collect the coefficients of the $\Sigma_\gamma Q_{\alpha\gamma}Q_{\beta\gamma}$ terms to get the new w (to be called $\tilde{w}$), that of the $Q^3_{\alpha\beta}$ terms to get the new u, etc. In addition, since the eigenvalue we seek is already of the order $(T-T')^2$, we can content ourselves with setting up this mapping for $T=T'$, i.e. all we have to do is to extend the mapping written up in De Dominicis *et al* (1991) to include the new couplings y and v. To carry out this calculation is fairly tedious but straightforward. We record only the result:

$$
\begin{aligned}
\tilde{\tau}&=\tau+\frac{1}{z}\{(2u-v+2y)I_2-2w^2I_4\}\\
\tilde{u}&=u+\frac{1}{z}\{(6u^2+3v^2+12y^2+6uv-24uy)I_4+(12vw^2+24yw^2-12uw^2)I_6+12w^4I_8\}\\
\tilde{y}&=y-\frac{1}{z}\{(10y^2-2v^2-4uy-6vy)I_4+(4uw^2-16vw^2-2yw)I_6-8w^4I_8\}\\
\tilde{v}&=v+\frac{1}{z}\{(v^2-8vy)I_4-8yw^2I_6\}\\
\tilde{w}&=w+\frac{1}{z}\{(-3vw-6yw)I_4-2w^3I_6\}
\end{aligned}
\tag{12}
$$

where

$$I_k=\int\frac{\mathrm{d}^dp}{(2\pi)^d}\frac{1}{p^k}.$$

The smallest squared 'mass' will then be given by an expression like (9) with v and w replaced by $\tilde{v}$ and $\tilde{w}$, respectively. To $O(1/z)$ and with the bare v and w put to 1 it reads:

$$\lambda_{\min}=\frac{1}{2z}(11I_4-4I_6)(T-T')^2. \tag{13}$$

For $d>8$ the prefactor is now positive, a finite mass obtains which means that the correlation function $\overline{\langle s_is_j\rangle_T\langle s_is_j\rangle_{T'}}$ falls off exponentially, with a characteristic length $\xi_{TT'}\sim\lambda_{\min}^{-1/2}\sim|T-T'|^{-1}$.

At $d=8$ the mapping (12) goes singular, the loop-integral I_8 blows up logarithmically. The origin and consequences of this singularity have been discussed by several authors (Green *et al* 1983, Fisher and Sompolinsky 1985, De Dominicis *et al* 1991, Kondor *et al* 1992) and are now well understood. The main message of these studies is that for $6<d<8$, in the standard, $T=T'$, situation the leading terms in all quantities of interest (phase boundary, masses, order parameter, etc) are correctly predicted by an 'effective' MFT in which the u coupling is replaced by $\tilde{u}=u+1/z$ constant $\times\, t^{(d/2)-4}$

and all the others are left at their bare values. This prescription cannot be taken over to $\lambda_{\min}$, because, in contrast to the usual cases where v and y enter subleading terms only, here v/w^2 determines the leading term itself.

Although the mapping (12) becomes singular at $d=8$, $\tilde{v}$ and $\tilde{w}$ remain well defined down to $d=6$. One might then be tempted to continue (13) to below $d=8$. This would be all wrong. At the next, $1/z^2$ order the $d=8$ singularity will show up also in $\tilde{v}$ and $\tilde{w}$ and, therefore, in order to find the correct exponent of $\lambda_{\min}$ for $6<d<8$ one should go to two-loop order. This is beyond the scope of the present letter.

To conclude, we have shown that, at least in high dimensions, the correlation overlap $\overline{\langle s_i s_j\rangle_T \langle s_i s_j\rangle_{T'}}$ is short ranged, with a characteristic length $\xi_{TT'} \sim |T-T'|^{-1}$. Technical complications prevented us from going below $d=8$ but we do not expect major surprises there: exponents will evidently change but we believe the exponential decay remains. Details of this work, including closed formulae for the correlation functions and covering also the magnetic case mentioned in the introduction will be published elsewhere.

One of the authors (IK) is obliged to C De Dominicis and G Parisi for valuable discussions and we greatly benefited from interaction with T Temesvári. This work was partially supported by the National Science Foundation OTKA Grant No 2090.

References

Alba M, Hamman J, Ocio M, Refregie Ph and Bouchiat H 1987 *J. Appl. Phys.* **61** 3683
Binder K and Young A P 1986 *Rev. Mod. Phys.* **58** 801
Bray A J and Moore M A 1987a *Glassy Dynamics and Optimization* ed J L van Hemmen and I Morgenstern (Berlin: Springer)
—— 1987b *Phys. Rev. Lett.* **58** 57
De Dominicis C and Kondor I 1983 *Phys. Rev.* B **27** 606
De Dominicis C, Kondor I and Temesvári T 1991 *J. Phys. A: Math. Gen.* **24** L301
Edwards S F and Anderson P W 1975 *J. Phys. F: Metal Phys.* **5** 965
Fisher D S and Huse D A 1986 *Phys. Rev. Lett.* **56** 1601
—— 1988 *Phys. Rev.* B **38** 373
Fisher D S and Sompolinsky H 1985 *Phys. Rev. Lett.* **54** 1063
Green J E, Moore M A and Bray A J 1983 *J. Phys. C: Solid State Phys.* **16** L815
Kondor I 1989 *J. Phys. A: Math. Gen.* **22** L163
Kondor I, De Dominicis C and Temesvári T 1992 *Physica* **185A** 295
Koper G J M and Hilhorst H J 1988 *J. Physique* **49** 429
Lundgren L, Svendlindh R, Nordblad P and Beckman O 1983 *Phys. Rev. Lett.* **51** 911
Mézard M, Parisi G and Virasoro M A 1987 *Spin Glass Theory and Beyond* (Singapore: World Scientific)
Parisi G 1979 *Phys. Rev. Lett.* **43** 1754
—— 1983 *Phys. Rev. Lett.* **50** 1946
Sherrington D and Kirkpatrick S 1975 *Phys. Rev. Lett.* **35** 1792